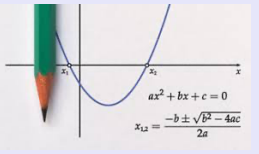
Math 125

Spring 2022

Lecture 8



Class QZ 5

1) Graph and shade

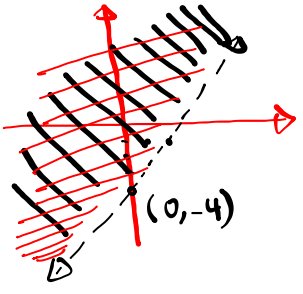
$3x-2y<8$

$-2y<-3x+8$

$y>\frac{-3}{-2}x+\frac{8}{-2}$

$y>\frac{3}{2}x-4$

Dotted line



2) Factor

$4x-36$

$4(x-9)$ ✓

Solve $|2x+5|-3=7$

$|2x+5|=10$

$2x+5=10$ OR $2x+5=-10$

$2x=5$ $\quad$ $2x=-15$

$x=\frac{5}{2}$ $\quad$ $x=\frac{-15}{2}$

$\left\{\frac{-15}{2}, \frac{5}{2}\right\}$

Solve $-3|4x-1|+5=-25$

Always Isolate the Abs. Value.

$-3|4x-1|=-25-5$

$-3|4x-1|=-30$

$|4x-1|=\frac{-30}{-3}$

$|4x-1|=10$

$4x-1=10$ OR $4x-1=-10$

$4x=11$ $\quad$ $4x=-9$

$x=\frac{11}{4}$ $\quad$ $x=\frac{-9}{4}$

Solution Set $\left\{\frac{-9}{4}, \frac{11}{4}\right\}$

Solve $|2x+7| = |x-3|$

$2x+7 = x-3$ OR $2x+7 = -(x-3)$

$2x - x = -3-7$

$x = -10$

$\boxed{x = -10}$

$2x+7 = -x+3$

$2x + x = 3-7$

$3x = -4$

$\boxed{x = \frac{-4}{3}}$

$\left\{-10, \frac{-4}{3}\right\}$

Solve $|3x-6| = |3x+6|$

$3x-6 = 3x+6$ OR $3x-6 = -(3x+6)$

$3x - 3x = 6+6$

$0 = 12$

False

$3x-6 = -3x-6$

$3x+3x = -6+6$

$6x = 0$

$x = \frac{0}{6}$ $\boxed{x=0}$

$\{0\}$

Do not use $\emptyset$ for Zero.

Solve $|2x+8| - |x+8| = 0$

Always Isolate Abs. Value.

$|2x+8| = |x+8|$

$2x+8 = x+8$ OR $2x+8 = -(x+8)$

$2x - x = 8 - 8$

$x = 0$

$2x+8 = -x-8$

$2x+x = -8-8$

$3x = -16$

$x = -\frac{16}{3}$

$\left\{-\frac{16}{3}, 0\right\}$

Abs. Value inequalities:

For $K > 0$

we want to Solve

$|ax+b| > K$ $|ax+b| < K$

$|ax+b| \geq K$ $|ax+b| \leq K$

1) Solve $|ax+b| = K$

2) Place Solutions on the number line System.

Shade outside of Solutions

Shade inside of Solutions

Final Ans:

1) Graphing

2) Interval notation

3) Set-Builder notation

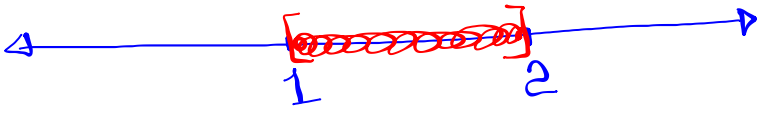
Solve $|2x-3| \leq 1$ ← Shade inside

Solve $|2x-3|=1$

$2x-3=1$ OR $2x-3=-1$

$2x=4$ $\quad 2x=2$

$x=2$ $\quad x=1$



I.N. $[1,2]$ S.B.N. $\{x \mid 1 \leq x \leq 2\}$

Solve $|3x+4| > 5$ ← Shade outside

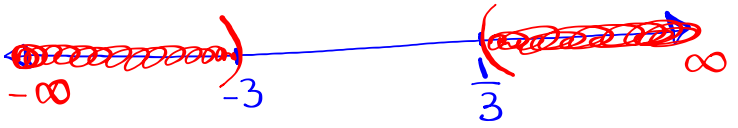
Solve $|3x+4|=5$

$3x+4=5$ OR $3x+4=-5$

$3x=1$ $\quad 3x=-9$

$x=\frac{1}{3}$ $\quad x=-3$

Graphing



I.N. $(-\infty,-3) \cup (\frac{1}{3},\infty)$ S.B.N.

$\{x \mid x<-3 \text{ OR } x>\frac{1}{3}\}$

Solve $|x+4|-2<5$

Always Isolate the Abs. Value.

NO =

$|x+4|<7$

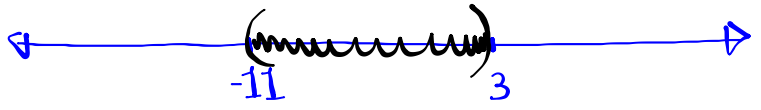
Shade between

Solve $|x+4|=7$

$x+4=7$ $\quad$ $x+4=-7$

$x=3$ $\quad$ $x=-11$

Graphing



I.N. $(-11,3)$ $\quad$ S.B.N. $\{x \mid -11<x<3\}$

Solve $-2|x-4|+3 \leq -7$

$-2|x-4| \leq -7-3$

$-2|x-4| \leq -10$

$\frac{-2}{-2}|x-4| \geq \frac{-10}{-2}$

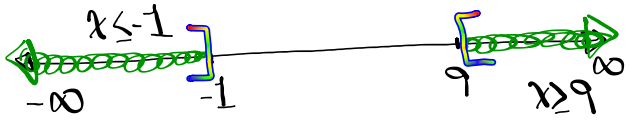
$|x-4| \geq 5$ shade outside

Always Isolate the Abs. Value

Solve $|x-4|=5$

$x-4=5$ OR $x-4=-5$

$x=9$ $\quad$ $x=-1$



I.N. $(-\infty,-1] \cup [9,\infty)$

S.B.N. $\{x \mid x \leq -1 \text{ OR } x \geq 9\}$

Class QZ 6

Solve $|2x-7|-3=0$ → $\{2,5\}$

$|2x-7|=3$

$2x-7=3$ OR $2x-7=-3$

$2x=10$ $2x=4$

$\boxed{x=5}$ $\boxed{x=2}$

Solve $|2x-1|+3<0$

$|2x-1|<-3$ No Solution

$\emptyset$

Do not do this $\{\emptyset\}$

that is a NO! NO!

Solve $-2|x+4|-8<0$

$-2|x+4|<8$

$|x+4|>\frac{8}{-2}$

$|x+4|>-4$

True

All Reals

$(-\infty,\infty)$

$\mathbb{R}$

Solve $\frac{1}{2}|x-1|-1 \geq 2$

Always Isolate the Abs. Value.

$\frac{1}{2}|x-1| \geq 3$

$|x-1| \geq 6$ — shade outside, use [or]

Solve $|x-1|=6$

$x-1=6$ OR $x-1=-6$

$x=7$ $x=-5$

I.N. $(-\infty, -5] \cup [7, \infty)$

S.B.N. $\{x \mid x \leq -5 \text{ OR } x \geq 7\}$

Solve $\frac{-2}{3}|x+2|+6 \geq 0$

Always Isolate the Abs. Value

$\frac{-2}{3}|x+2| \geq -6$

Multiply by 3

$-2|x+2| \geq -18$

Divide by -2

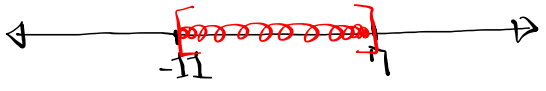
$|x+2| \leq \frac{-18}{-2}$

$|x+2| \leq 9$ — shade between, use [,]

Solve $|x+2|=9$

$x+2=9$ $x+2=-9$

$x=7$ $x=-11$



I.N. $[-11, 7]$, S.B.N. $\{x \mid -11 \leq x \leq 7\}$

Factor Completely:

$A^2 - B^2 = (A+B)(A-B)$

$A^2 + B^2 = \text{Prime}$

1) $4x - 100 = \boxed{4(x-25)}$

2) $4x^2 - 100 = 4(x^2 - 25) = 4(x^2 - 5^2)$

$A^2 - B^2$

$= \boxed{4(x+5)(x-5)}$

3) $4x^3 - 32$

$= 4(x^3 - 8) = 4(x^3 - 2^3) = \boxed{4(x-2)(x^2+2x+4)}$

$A^3 - B^3 = (A-B)(A^2 + AB + B^2)$

$A^3 + B^3 = (A+B)(A^2 - AB + B^2)$

Factor Completely:

1) $x^2 - 100 = x^2 - 10^2 = \boxed{(x+10)(x-10)}$

2) $x^2 + 81 = x^2 + 9^2 = \boxed{\text{Prime}}$

3) $x^3 + 64 = x^3 + 4^3 = (x+4)(x^2 - 4x + 16)$

4) $x^3 - 125 = x^3 - 5^3 = (x-5)(x^2 + 5x + 25)$

Zero – Factor Property:

Zero – Product Rule:

If $A \cdot B = 0$, then $A = 0$ OR $B = 0$ (Maybe both)

Solve $(x-3)(x+8) = 0$

$x - 3 = 0$ OR $x + 8 = 0$

$x = 3$ $\quad$ $x = -8$

$\{-8, 3\}$

Solve $(x-5)(x+5)(x-10) = 0$

Is RHS = 0? Yes

Is LHS Factored? Yes

$\Rightarrow$ Use Zero-Product rule to Solve

$x - 5 = 0$ $\quad$ $x + 5 = 0$ $\quad$ $x - 10 = 0$

$x = 5$ $\quad$ $x = -5$ $\quad$ $x = 10$

$\{\pm 5, 10\}$

Use Zero-Factor property to Solve

$(2x-5)(3x+4)=0.$

$2x-5=0$

$2x=5$

$x=\frac{5}{2}$

$3x+4=0$

$3x=-4$

$x=\frac{-4}{3}$

$\left\{\frac{-4}{3}, \frac{5}{2}\right\}$

Solve $\frac{1}{3}x - \frac{1}{2} = \frac{5}{6}$

Use LCD to clear fractions

LCD = 6

$\cancel{6}^{2} \cdot \frac{1}{\cancel{3}}x - \cancel{6}^{3} \cdot \frac{1}{\cancel{2}} = \cancel{6} \cdot \frac{5}{\cancel{6}}$

$2x - 3 = 5$

$2x = 8$

$x = 4$

$\{4\}$

Solve $\frac{2}{x-3} - \frac{1}{x+2} = \frac{5}{x^2-x-6}$

$LCD = (x-3)(x+2)$

$(x-3)(x+2)\cdot\frac{2}{x-3} - (x-3)(x+2)\cdot\frac{1}{x+2} = (x-3)(x+2)\cdot\frac{5}{x^2-x-6}$

$2(x+2) - 1(x-3) = 5$

$2x+4 - x + 3 = 5$

$x + 7 = 5$

$x = -2$

No Solution

-2 makes at least one of the denominators Zero.

$\varnothing$

Class QZ 7

Factor Completely

1) $x^2 - x - 6 = (x-3)(x+2)$

2) $x^3 + 1000 = x^3 + 10^3 = (x+10)(x^2-10x+100)$

3) $3x^2 - 75x = 3x(x-25)$